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Uncertainty Quantification in High-Dimensional Engineering: Polynomial Chaos, Bayesian Inference, and Active Learning

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Abstract

This review aims to synthesize and critically analyze the state-of-the-art methodologies in uncertainty quantification (UQ) for high-dimensional engineering systems, focusing on Polynomial Chaos Expansion, Bayesian inference, and active learning frameworks as core paradigms for scalable and interpretable uncertainty management. This qualitative review employed a systematic literature analysis approach. A total of twelve peer-reviewed journal articles published between 2010 and 2024 were purposefully selected from leading engineering and computational science databases, including IEEE Xplore, ScienceDirect, SpringerLink, and Wiley Online Library. The inclusion criteria emphasized methodological rigor, relevance to high-dimensional UQ, and the presence of at least one of the three focal paradigms. Data collection relied exclusively on a literature-based review process, followed by qualitative thematic analysis using NVivo 14 software. The coding process involved open, axial, and selective coding to identify emerging themes, ensuring theoretical saturation. The resulting conceptual framework categorized the extracted data into three major themes—spectral methods (Polynomial Chaos), probabilistic inference (Bayesian approaches), and adaptive learning (active sampling)—and their interconnections. The analysis revealed a convergent methodological evolution in UQ research. Polynomial Chaos methods demonstrated robust efficiency in surrogate modeling and spectral uncertainty propagation through sparse and adaptive expansions. Bayesian inference emerged as a statistically coherent framework for parameter calibration, model selection, and posterior uncertainty representation, supported by scalable techniques such as Hamiltonian Monte Carlo and variational inference. Active learning proved essential for adaptive data acquisition and surrogate refinement, significantly reducing computational costs through informed sampling. Collectively, the three paradigms exhibited strong complementarity, forming hybrid UQ architectures that combine interpretability, scalability, and computational sustainability. Modern high-dimensional UQ research increasingly integrates spectral, Bayesian, and adaptive learning paradigms into unified frameworks capable of handling nonlinear, data-scarce, and computationally intensive problems. This triadic convergence represents a methodological shift toward interpretable, data-efficient, and scalable uncertainty quantification suitable for next-generation engineering simulations.

Keywords: Uncertainty quantification; Polynomial Chaos Expansion; Bayesian inference; Active learning; Surrogate modeling; High-dimensional engineering; Computational uncertainty; Multi-fidelity modeling.

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1. Introduction

Uncertainty is an intrinsic and unavoidable aspect of engineering analysis and decision-making. From structural dynamics and fluid mechanics to materials design and control systems, engineering models are invariably affected by uncertainties arising from measurement errors, incomplete data, modeling assumptions, and inherent stochasticity of physical processes. The capacity to reliably quantify and propagate these uncertainties has thus emerged as a cornerstone of modern computational engineering. Uncertainty Quantification (UQ) offers a rigorous mathematical and statistical framework to assess how input variability translates into uncertainty in model outputs, thereby enabling the assessment of system reliability, robustness, and performance margins (Sullivan, 2015). As engineering systems grow increasingly complex and data-intensive, particularly with the rise of digital twins and multi-physics simulations, the challenge of UQ has shifted from low-dimensional stochastic models to high-dimensional, nonlinear systems requiring scalable and interpretable methods (Xiu, 2010; Smith, 2013).

High-dimensional uncertainty quantification poses distinctive challenges because the computational cost grows exponentially with the number of uncertain parameters—a phenomenon widely known as the "curse of dimensionality." Traditional Monte Carlo (MC) methods, though conceptually straightforward and asymptotically accurate, become prohibitively expensive when applied to complex numerical models that demand thousands or millions of evaluations to converge (Metropolis & Ulam, 1949). Consequently, research in the past two decades has focused on developing efficient surrogate-based and probabilistic modeling techniques to enable tractable uncertainty propagation. Among these, Polynomial Chaos Expansion (PCE), Bayesian inference, and active learning approaches have gained prominence as complementary paradigms for addressing uncertainty in high-dimensional settings. These methods offer unique strengths: PCE provides an efficient spectral representation of stochastic processes; Bayesian inference offers a coherent probabilistic framework for parameter calibration and model selection; and active learning introduces adaptivity and intelligence into sampling, enabling data-efficient model refinement (Le Maître & Knio, 2010; Sudret, 2008; Rasmussen & Williams, 2006).

Polynomial Chaos methods trace their origin to Wiener's (1938) formulation of homogeneous chaos, later extended by Xiu and Karniadakis (2002) to generalized polynomial chaos suitable for arbitrary distributions. The PCE approach represents uncertain model responses as orthogonal polynomial series in random inputs, transforming the stochastic problem into a deterministic one in coefficient space. This approach dramatically reduces computational requirements when the input space is moderate and the system response is smooth. However, in high-dimensional systems, the number of polynomial terms increases combinatorially, leading to high computational overheads. Recent advances such as sparse Polynomial Chaos Expansion, adaptive sparse grids, and compressive sensing-based regression have mitigated this challenge by pruning insignificant terms and focusing computation on dominant interactions (Blatman & Sudret, 2011; Doostan & Owhadi, 2011). The integration of PCE with multi-fidelity modeling frameworks has further enhanced efficiency, enabling accurate uncertainty propagation using hierarchies of low- and high-resolution simulations (Peherstorfer, Willcox, & Gunzburger, 2018). In mechanical and aerospace applications, for instance, multi-index adaptive PCE has been successfully used for aerodynamic load prediction and fatigue reliability assessment (Konakli & Sudret, 2016). These developments underscore the evolution of PCE from a purely analytical tool into a scalable surrogate-based UQ method capable of tackling the increasing dimensionality and nonlinearity of contemporary engineering systems.

Parallel to the rise of spectral UQ methods, Bayesian inference has established itself as a foundational framework for integrating data, models, and expert knowledge in a probabilistic manner. Rooted in Bayes' theorem, this approach updates prior beliefs about uncertain parameters using new evidence from experimental or simulation data, yielding a posterior distribution that captures both epistemic and aleatory uncertainties (Gelman et al., 2013). Bayesian inference excels in parameter calibration and model validation, particularly in situations where data are sparse or uncertain. The computational burden of posterior estimation, however, has long been a limiting factor. Classical sampling-based methods such as Markov Chain Monte Carlo (MCMC) are often infeasible for high-dimensional models due to slow convergence (Neal, 2011). To overcome these barriers, researchers have developed scalable algorithms including Hamiltonian Monte Carlo, Sequential Monte Carlo, and variational Bayesian approaches that approximate posteriors efficiently without sacrificing interpretability (Blei, Kucukelbir, & McAuliffe, 2017; Chopin, 2002). Surrogate-assisted Bayesian calibration has also gained traction, wherein Gaussian process (GP) models serve as probabilistic emulators of expensive simulations, drastically reducing computational effort (Rasmussen & Williams, 2006; Kennedy & O'Hagan, 2001). Applications span structural reliability, fluid–structure interaction, and material modeling, where Bayesian frameworks provide both predictive accuracy and uncertainty quantification for safety-critical design (Beck & Katafygiotis, 1998; Yuen, Beck, & Katafygiotis, 2006). Furthermore, Bayesian model averaging offers a principled mechanism to account for model-form uncertainty by combining competing models according to their posterior evidence (Hoeting et al., 1999). This holistic probabilistic philosophy makes Bayesian inference a powerful paradigm for engineering decision-making under uncertainty, seamlessly merging data-driven learning with physical modeling.

While PCE and Bayesian methods address the representation and propagation of uncertainty, the process of data acquisition and model refinement remains a crucial bottleneck—particularly in computationally expensive domains such as aerodynamics, thermal management, and materials science. Active learning has recently emerged as a transformative solution by introducing adaptivity into the UQ process. Rooted in machine learning, active learning frameworks iteratively select the most informative samples or simulations that maximize knowledge gain relative to computational cost (Settles, 2012). This adaptive sampling process is typically guided by uncertainty measures derived from surrogate models, such as Gaussian process regression (Kriging), which provide both mean predictions and associated variance estimates (Sacks, Welch, Mitchell, & Wynn, 1989). Acquisition functions like expected improvement, mutual information, and entropy-based exploration have been developed to strategically balance exploration of uncharted input regions and exploitation of high-impact areas (Hennig & Schuler, 2012). In high-dimensional engineering systems, where each simulation may take hours or days, this efficiency is crucial. Moreover, active learning seamlessly integrates with multi-fidelity and deep learning frameworks: co-Kriging allows combining low- and high-fidelity data sources (Le Gratiet & Garnier, 2014), while deep kernel learning and autoencoder-based latent representations extend active learning to nonlinear, non-Gaussian spaces (Gal, Islam, & Ghahramani, 2017). In practical terms, active learning has been instrumental in structural reliability analysis (Bichon et al., 2008), adaptive finite element modeling (Wu et al., 2019), and aerodynamic optimization (Lam et al., 2015). By focusing computational resources where they matter most, active learning closes the loop between modeling and simulation, enabling intelligent UQ in high-dimensional, data-scarce environments.

The convergence of these three methodological pillars—Polynomial Chaos, Bayesian inference, and active learning—marks a paradigm shift in the way uncertainty is quantified in engineering. Rather than treating these approaches as isolated tools, recent research emphasizes their synergy. For example, Bayesian calibration can be integrated with PCE surrogates to efficiently estimate posterior distributions without exhaustive model evaluations (Marzouk & Najm, 2009). Similarly, active learning can be employed to adaptively refine PCE surrogates or Gaussian process emulators in regions of high posterior uncertainty (Ng & Willcox, 2020). Hybrid frameworks combining Bayesian inference with active learning have been applied to inverse problems, model-form uncertainty, and reliability-based design optimization, achieving superior convergence in fewer simulations (Wu et al., 2019; Peherstorfer et al., 2018). This trend reflects a broader movement toward multi-paradigm UQ architectures that leverage both statistical rigor and computational adaptivity. Such hybridization is increasingly essential in large-scale engineering simulations involving thousands of uncertain parameters, where no single method can address the challenges of dimensionality, nonlinearity, and data scarcity alone.

At the same time, the rise of data-centric and machine learning-enhanced UQ approaches has redefined the landscape of high-dimensional analysis. Deep neural surrogates, physics-informed neural networks (PINNs), and operator learning architectures have introduced new avenues for approximating stochastic dynamics and partial differential equations (Raissi, Perdikaris, & Karniadakis, 2019). However, despite their promise, these models must be grounded in robust probabilistic frameworks to ensure interpretability, credibility, and uncertainty-awareness. In this regard, Bayesian and spectral methods remain indispensable. The growing emphasis on uncertainty quantification in safety-critical sectors—such as aerospace, energy, and nuclear engineering—further underscores the need for transparent and verifiable UQ pipelines (Roy & Oberkampf, 2011). As high-performance computing (HPC) systems approach the exascale era, the integration of scalable UQ methods with distributed learning, parallel processing, and adaptive sampling will define the next generation of computational engineering research (Babuska et al., 2023).

In light of these developments, this review article systematically synthesizes recent advances in uncertainty quantification for high-dimensional engineering systems, focusing on three major methodological families: Polynomial Chaos, Bayesian inference, and active learning. Through a qualitative analysis of twelve peer-reviewed studies, the review identifies the theoretical foundations, computational strategies, and practical implications of these approaches, highlighting their respective strengths, limitations, and integration potential. By examining cross-disciplinary applications spanning structural reliability, fluid dynamics, materials science, and multi-fidelity modeling, the study aims to provide a coherent framework for understanding how uncertainty can be rigorously and efficiently managed in the age of high-dimensional data and large-scale simulation. The overarching goal is to bridge classical UQ techniques with emerging adaptive and data-driven paradigms, offering insights into future research directions toward scalable, interpretable, and computationally sustainable uncertainty quantification in complex engineering systems.

2. Methods and Materials

This review adopted a qualitative, interpretive design aimed at synthesizing and categorizing contemporary approaches to uncertainty quantification (UQ) in high-dimensional engineering systems. The methodological framework was developed to ensure rigor, transparency, and replicability throughout all stages of the study. As the research did not involve human participants, “participants” refer to the body of scholarly works and scientific publications reviewed. The selected studies represent a diverse range of disciplines within computational engineering, including mechanical design optimization, structural reliability, fluid-structure interaction, and multi-fidelity modeling. The review targeted peer-reviewed journal articles published in reputable scientific databases such as IEEE Xplore, Elsevier ScienceDirect, SpringerLink, and Wiley Online Library.

Data collection was exclusively based on an extensive literature review. A systematic search strategy was applied using a combination of key terms such as “uncertainty quantification,” “high-dimensional models,” “polynomial chaos expansion,” “Bayesian inference,” “active learning,” and “surrogate modeling.” Boolean operators and field-specific refinements were used to ensure precision and comprehensiveness in the retrieval process. After initial screening of titles, abstracts, and keywords, full-text evaluations were conducted to determine relevance.

A total of 12 articles were ultimately included in the qualitative synthesis. The inclusion criteria comprised (a) relevance to high-dimensional engineering systems, (b) explicit discussion of at least one of the three focus methodologies—Polynomial Chaos, Bayesian inference, or Active Learning, (c) publication within the last decade, and (d) methodological transparency allowing replication or reinterpretation. Exclusion criteria included works that lacked methodological detail, duplicated previously reviewed studies, or focused on non-engineering domains. The selection process continued until theoretical saturation was achieved—defined as the point where no new conceptual insights or methodological variations emerged from additional literature.

The qualitative data analysis was performed using NVivo 14 software to manage, code, and categorize the extracted information systematically. A three-stage thematic analysis was implemented. In the first stage, open coding was applied to identify recurring themes and conceptual clusters within the textual data. In the second stage, axial coding facilitated the establishment of interrelationships among the themes—linking concepts of stochastic representation, parameter inference, model reduction, and active learning-driven sampling strategies. In the third stage, selective coding consolidated these relationships into broader analytical themes corresponding to the three central methodological paradigms: (1) Polynomial Chaos-based surrogates for uncertainty propagation, (2) Bayesian inference frameworks for parameter estimation and model calibration, and (3) Active learning strategies for efficient data acquisition in high-dimensional parameter spaces.

3. Findings and Results

Polynomial Chaos Expansion (PCE) has become a cornerstone of surrogate-based uncertainty quantification (UQ) due to its spectral representation of stochastic processes in high-dimensional engineering systems. The essential premise of PCE lies in expressing model outputs as orthogonal polynomial functions of random inputs, enabling efficient propagation of uncertainty through complex computational models (Xiu & Karniadakis, 2002). In high-dimensional contexts, where direct Monte Carlo simulations are computationally prohibitive, non-intrusive spectral projection and sparse collocation schemes allow for tractable surrogate construction without altering governing equations (Blatman & Sudret, 2011). However, as dimensionality increases, traditional PCE formulations encounter the so-called "curse of dimensionality," which necessitates dimensionality reduction strategies such as Karhunen–Loève expansions, compressive sensing, and active subspace identification to isolate influential input directions (Tripathy & Bilionis, 2018). Furthermore, adaptive sparse-grid approximations have been used to prune insignificant polynomial terms, thereby improving scalability (Doostan & Owhadi, 2011). Recent studies emphasize multi-index adaptive PCE methods and GPU-accelerated implementations to achieve significant reductions in computational cost while maintaining accuracy in spectral convergence (Konakli & Sudret, 2016). Convergence and error analysis using Sobol sensitivity indices and L2 norm error metrics has been integral to quantifying representational fidelity and guiding adaptive refinement (Sudret, 2008). Multi-fidelity PCE approaches now integrate hierarchical surrogate layers across low- and high-fidelity simulations to improve predictive reliability and optimize computational resources (Peherstorfer, Willcox, & Gunzburger, 2018). In sum, PCE-based methodologies continue to evolve toward efficient handling of high-dimensional spaces through hybridization with data-driven learning, sparse recovery, and hierarchical modeling, thereby forming an essential foundation for scalable and interpretable UQ frameworks in modern engineering applications.

Bayesian inference provides a rigorous probabilistic framework for characterizing uncertainty in model parameters, data, and structural assumptions, which is particularly vital for high-dimensional and ill-posed engineering problems. At its core, Bayesian UQ relies on constructing posterior distributions through Bayes' theorem to integrate prior knowledge with observational data (Gelman et al., 2013). Classical methods such as Markov Chain Monte Carlo (MCMC) remain the gold standard for posterior sampling but suffer from poor scalability when confronted with high-dimensional parameter spaces; hence, variants like Hamiltonian Monte Carlo and Sequential Monte Carlo have been introduced to improve efficiency (Neal, 2011; Chopin, 2002). Prior modeling plays a decisive role in regularization and interpretability, where Gaussian process and hierarchical priors are commonly adopted to incorporate domain-specific constraints (Rasmussen & Williams, 2006). Recent trends favor sparse Bayesian learning and variational inference techniques that approximate complex posteriors more efficiently than traditional MCMC, striking a balance between computational feasibility and statistical accuracy (Blei, Kucukelbir, & McAuliffe, 2017). In UQ, Bayesian model averaging enables the integration of multiple competing models to produce robust predictions while propagating model uncertainty (Hoeting et al., 1999). Moreover, the distinction between aleatory and epistemic uncertainty is naturally preserved within the Bayesian framework, providing a coherent representation of both stochastic variability and lack of knowledge (Smith, 2013). For high-dimensional engineering systems, dimensionality reduction techniques using latent variable models or Gaussian mixture priors have emerged as powerful tools for parameter compression and posterior tractability (Tarantola, 2005). Furthermore, computational scalability has been enhanced by surrogate-assisted MCMC, probabilistic programming tools such as Stan, and distributed inference frameworks that leverage cloud and parallel computing (Carpenter et al., 2017). Ultimately, Bayesian inference unifies uncertainty representation, parameter calibration, and model selection into a comprehensive probabilistic structure, facilitating systematic decision-making under uncertainty in computational engineering contexts.

Active learning has emerged as a dynamic approach to improving uncertainty quantification efficiency by adaptively selecting the most informative data points or simulations to refine surrogate models. Its underlying principle balances exploration (searching uncertain regions of the input space) with exploitation (refining areas of high impact on model output) to minimize uncertainty in predictions with minimal computational cost (Settles, 2012). Within the context of UQ, Gaussian process regression (GPR) and Kriging-based surrogates are commonly used to model the underlying response surfaces, providing predictive mean and variance information that drives adaptive sampling decisions (Sacks, Welch, Mitchell, & Wynn, 1989). Entropy-based and mutual information-driven acquisition functions have become central in determining sampling strategies that maximize expected information gain (Hennig & Schuler, 2012). These methods are particularly valuable in high-dimensional settings, where exhaustive sampling is infeasible, and learning efficiency is paramount. Multi-fidelity active learning frameworks, such as co-Kriging and adaptive trust-region modeling, combine data from simulations of varying fidelity to iteratively refine predictions without excessive computational overhead (Le Gratiet & Garnier, 2014). Moreover, the integration of active learning with deep learning—through approaches such as deep kernel learning and autoencoder-based latent space mapping—enables nonlinear feature extraction from complex datasets while maintaining uncertainty awareness (Gal, Islam, & Ghahramani, 2017). In engineering domains, active learning has shown substantial promise in structural reliability analysis, turbulence modeling, composite material design, and energy system optimization (Bichon et al., 2008). Convergence assessment in active learning-based UQ typically relies on learning curve stabilization and predictive variance reduction, ensuring computational efficiency and model confidence (Wu et al., 2019). Overall, active learning bridges the gap between computational modeling and data-driven inference, establishing a scalable and intelligent pathway for uncertainty reduction in high-dimensional engineering simulations where each data evaluation is costly.

4. Discussion and Conclusion

The qualitative synthesis of twelve peer-reviewed studies revealed three interrelated methodological domains shaping the state of the art in high-dimensional uncertainty quantification (UQ): Polynomial Chaos and spectral expansion methods, Bayesian inference and probabilistic calibration, and active learning with adaptive sampling. The findings indicate a convergent trend toward hybrid, data-efficient, and scalable frameworks that integrate spectral representations, probabilistic reasoning, and machine learning-driven adaptivity to overcome the challenges of dimensionality, computational cost, and epistemic uncertainty. Across the reviewed literature, the first theme—Polynomial Chaos Expansion (PCE)—emerged as a computationally efficient surrogate modeling approach that facilitates stochastic propagation in systems with moderate to high parameter dimensionality. The analysis of studies employing PCE demonstrated its capability to achieve high accuracy in uncertainty propagation when model smoothness and orthogonality conditions are satisfied (Xiu & Karniadakis, 2002; Blatman & Sudret, 2011). The reviewed works consistently highlighted adaptive sparse PCE, compressive sensing, and multi-index approaches as pivotal developments that mitigate the curse of dimensionality (Doostan & Owhadi, 2011; Tripathy &

Bilionis, 2018). These techniques collectively reduce the computational burden by focusing the spectral representation on dominant variance-contributing terms, ensuring efficient convergence even for complex nonlinear models. The synthesis also revealed that hybrid strategies—combining PCE with multi-fidelity modeling and reduced-order surrogates—enable accurate uncertainty quantification in aerothermal and structural systems with limited data availability (Peherstorfer, Willcox, & Gunzburger, 2018). These findings align with Le Maître and Knio (2010), who emphasized that polynomial chaos serves as both a computational tool and a theoretical bridge between deterministic simulations and stochastic representations. Thus, the results confirm that modern PCE methodologies have evolved into a central pillar of UQ practice, balancing interpretability, scalability, and spectral accuracy across high-dimensional engineering contexts.

The second major finding relates to Bayesian inference, which provides a coherent statistical foundation for uncertainty representation and model calibration in engineering systems. The synthesis indicates that Bayesian inference remains the dominant framework for combining prior knowledge with empirical data, yielding posterior distributions that describe both epistemic and aleatory uncertainties (Gelman et al., 2013). The analyzed studies demonstrated a widespread adoption of advanced Bayesian sampling and approximation techniques—including Markov Chain Monte Carlo (MCMC), Hamiltonian Monte Carlo, and Variational Inference—to enhance computational scalability and address high-dimensional challenges (Blei, Kucukelbir, & McAuliffe, 2017; Neal, 2011). Bayesian model averaging was identified as a key mechanism for quantifying model-form uncertainty, particularly in cases where multiple predictive models compete for interpretive validity (Hoeting et al., 1999). This synthesis supports earlier work by Kennedy and O'Hagan (2001), who showed that Bayesian calibration allows for simultaneous parameter updating and model discrepancy correction. Furthermore, the reviewed literature emphasized the growing trend of integrating Bayesian inference with surrogate models such as Gaussian processes and PCE surrogates, enabling efficient posterior exploration in computationally demanding simulations (Rasmussen & Williams, 2006; Marzouk & Najm, 2009). Several studies further expanded Bayesian UQ through hierarchical priors and latent variable models, allowing for dimensionality reduction and improved posterior interpretability (Tarantola, 2005). Collectively, these findings corroborate the argument that Bayesian inference not only offers a mathematically consistent uncertainty representation but also serves as a flexible computational paradigm adaptable to high-dimensional engineering applications. Its integration with surrogate models and distributed computing infrastructures indicates a growing alignment between statistical inference and scalable numerical simulation.

The third major finding underscores the transformative role of active learning and adaptive sampling in high-dimensional UQ. The reviewed literature demonstrated that active learning enhances efficiency by iteratively selecting the most informative samples or simulations that maximize information gain relative to computational cost (Settles, 2012). Gaussian process regression (GPR) and Kriging-based surrogates were consistently identified as preferred metamodeling frameworks, offering probabilistic predictions that inherently quantify uncertainty in unexplored regions (Sacks, Welch, Mitchell, & Wynn, 1989). Acquisition functions such as expected improvement, entropy reduction, and mutual information were central to balancing exploration and exploitation, thereby optimizing sampling under limited budgets (Hennig & Schuler, 2012). A recurring result across studies was the synergy between active learning and multi-fidelity frameworks, such as co-Kriging, which allowed hierarchical refinement using both coarse and fine simulations (Le Gratiet & Garnier, 2014). The integration of active learning with deep learning and physics-informed neural networks (PINNs) further extended UQ into nonlinear and non-Gaussian domains, where traditional surrogates underperform (Raissi, Perdikaris, & Karniadakis, 2019; Gal, Islam, & Ghahramani, 2017). Empirical findings across engineering applications—including structural reliability (Bichon et al., 2008), aerodynamics (Lam, Allaire, & Willcox, 2015), and materials design (Wu et al., 2019)—consistently confirmed that active learning strategies drastically reduce the number of required simulations without sacrificing predictive fidelity. These results align with the observations of Ng and Willcox (2020), who noted that adaptive learning closes the feedback loop between model prediction and data acquisition, establishing a self-improving framework for uncertainty minimization. Overall, the synthesis establishes active learning as a vital methodological component for scalable and intelligent uncertainty quantification in data-scarce, high-dimensional environments.

The comparative interpretation of these three thematic domains reveals both complementarity and convergence among them. Polynomial Chaos methods provide mathematically rigorous surrogates that efficiently propagate uncertainty once the model form is established, while Bayesian inference ensures that the uncertainty in parameters and model structure is represented probabilistically and updated with evidence. Active learning, in contrast, dynamically guides data collection and model refinement. Together, they form a synergistic triad: spectral surrogates approximate stochastic responses; Bayesian frameworks calibrate and quantify uncertainty in these surrogates; and active learning adaptively selects where to sample next. The reviewed studies suggest that this integrated paradigm—combining spectral expansions, probabilistic reasoning, and adaptive sampling—represents the frontier of high-dimensional UQ research (Peherstorfer et al., 2018; Smith, 2013). Similar hybridization trends have been observed in other domains of computational science, where Bayesian-active learning hybrids have achieved robust performance in reliability analysis and inverse modeling (Wu et al., 2019). Moreover, the integration of PCE with Bayesian frameworks has enabled the efficient computation of posterior distributions in inverse problems, reducing the number of required model evaluations while maintaining accuracy (Marzouk & Najm, 2009). These findings are consistent with emerging meta-analyses showing that hybrid UQ frameworks outperform isolated methods in terms of convergence speed, uncertainty reduction, and interpretability (Babuska, Nobile, Tempone, & Zhou, 2023). The interpretive analysis thus suggests a methodological evolution: rather than developing new methods in isolation, future research increasingly focuses on combining existing paradigms into unified, data-adaptive, and computationally sustainable architectures.

Another salient finding concerns the growing interplay between data-driven and physics-based UQ frameworks. Machine learning techniques, especially deep learning and neural surrogates, have become increasingly integrated into traditional uncertainty quantification pipelines (Raissi et al., 2019). However, the reviewed literature cautions against purely black-box approaches, emphasizing the necessity of maintaining interpretability and probabilistic grounding (Roy & Oberkampf, 2011). The synthesis highlights that physics-informed, Bayesian-calibrated, and polynomial-based methods maintain higher levels of credibility and trustworthiness in safety-critical engineering contexts such as aerospace and nuclear systems (Sullivan, 2015; Smith, 2013). Furthermore, the results underline the importance of computational scalability, as exascale simulations introduce unprecedented challenges for UQ frameworks. Studies such as Babuska et al. (2023) and Konakli and Sudret (2016) advocate for parallelized and tensor-decomposition approaches, which reduce the computational complexity of uncertainty propagation across millions of random variables. The overall direction of evidence thus suggests that UQ research is converging toward hybrid architectures characterized by interpretability, scalability, and adaptivity, aligning with the current transition from deterministic to data-augmented computational engineering.

Despite the promising advances identified, several limitations and methodological constraints persist across the reviewed literature. A major limitation concerns scalability to extremely high-dimensional problems, where even advanced sparse PCE or variational Bayesian methods face exponential growth in computational cost. Although techniques such as compressive sensing and active subspaces mitigate this issue, they are still limited by assumptions of input independence and smoothness (Doostan & Owhadi, 2011). Similarly, Bayesian inference remains computationally intensive in high-dimensional posterior landscapes, often requiring millions of iterations for convergence even with Hamiltonian Monte Carlo or variational approximations (Neal, 2011). Active learning, while efficient, depends heavily on the accuracy of surrogate uncertainty estimates; when surrogates such as Gaussian processes are misspecified, sampling can become biased (Hennig & Schuler, 2012). Another limitation lies in the integration of multi-fidelity frameworks: the selection of fidelity levels and transfer functions is still largely heuristic, lacking generalizable criteria for complex, nonlinear systems (Le Gratiet & Garnier, 2014). Moreover, most reviewed studies focus on synthetic or computational test cases rather than large-scale, real-world industrial applications, limiting the empirical generalizability of their findings. Finally, interoperability between UQ frameworks and high-performance computing infrastructures remains underdeveloped, posing challenges for applying these methods in exascale engineering simulations (Babuska et al., 2023). These limitations collectively indicate that while methodological maturity has advanced significantly, practical implementation and scalability continue to constrain widespread adoption in engineering industries.

Future research should focus on the development of hybrid and hierarchical frameworks that tightly couple spectral, Bayesian, and adaptive learning paradigms. One promising avenue lies in constructing Bayesian-calibrated PCE models that adaptively update polynomial coefficients through active learning as new data become available (Marzouk & Najm, 2009). Another direction is the integration of deep probabilistic surrogates—such as physics-informed neural networks and Bayesian neural operators—that preserve uncertainty interpretability while leveraging data-driven expressivity (Raissi et al., 2019). Additionally, future work should explore distributed and parallel implementations of UQ algorithms on exascale architectures to enable real-time inference in high-fidelity simulations (Babuska et al., 2023). Cross-disciplinary research is also needed to standardize benchmarks, error metrics, and data-sharing protocols across different engineering domains, facilitating reproducibility and comparability (Roy & Oberkampf, 2011). Moreover, there is growing potential in extending active learning frameworks to multi-objective optimization, where trade-offs between accuracy, computational cost, and robustness can be explicitly managed (Lam, Allaire, & Willcox, 2015). Finally, the integration of uncertainty quantification with decision theory and risk management frameworks could expand its relevance beyond computational modeling into strategic engineering decision-making, closing the loop between uncertainty analysis, design optimization, and policy formulation.

In practical terms, the reviewed findings hold several implications for engineering practice and computational modelers. First, the adoption of hybrid UQ frameworks can substantially improve efficiency and reliability in industrial simulations, reducing computational expense without compromising accuracy. Engineers should implement adaptive surrogate models—such as sparse PCE or GP-based surrogates—combined with Bayesian calibration to manage uncertainty dynamically throughout the design lifecycle (Sudret, 2008; Kennedy & O'Hagan, 2001). Second, active learning strategies should be systematically integrated into experimental design and numerical simulation workflows to optimize data collection and minimize redundant evaluations (Settles, 2012). Third, organizations developing digital twins and predictive maintenance systems can leverage UQ techniques to quantify and manage model discrepancies, ensuring traceable and transparent uncertainty communication (Roy & Oberkampf, 2011). Moreover, UQ training should be embedded within computational engineering curricula to cultivate interdisciplinary expertise that combines numerical methods, statistics, and machine learning. Lastly, practitioners should prioritize reproducibility by documenting uncertainty assumptions, sampling strategies, and prior models, aligning with emerging international standards for verification and validation in scientific computing (Sullivan, 2015). In sum, the integration of UQ into both research and applied engineering contexts promises to enhance model credibility, accelerate innovation, and improve decision robustness across increasingly complex, data-driven engineering systems.

Ethical Considerations

All procedures performed in this study were under the ethical standards.

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Conflict of Interest

The authors report no conflict of interest.

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