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Scientific Machine Learning for PDEs: Operators, Surrogates, and Error-Controlled Multi-Fidelity Schemes

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Abstract

This review aims to synthesize recent advances in scientific machine learning (SciML) for solving partial differential equations (PDEs), focusing on operator learning, surrogate modeling, and error-controlled multi-fidelity frameworks that integrate data-driven intelligence with physical consistency. This study adopted a qualitative, interpretive review design based on a systematic literature analysis of thirteen peer-reviewed journal articles published between 2019 and 2025. The data collection process relied exclusively on scholarly databases such as Scopus, ScienceDirect, and IEEE Xplore, targeting works addressing neural operator architectures, hybrid physics–ML couplings, and multi-fidelity adaptation. All sources were imported into Nvivo 14 software for coding and thematic synthesis. Open, axial, and selective coding cycles were performed until theoretical saturation was achieved. Four main categories—operator learning paradigms, surrogate and reduced-order models, error-controlled multi-fidelity schemes, and computational integration—were extracted and structured according to their conceptual relationships and methodological contributions. The review identified that operator learning (e.g., DeepONet, Fourier Neural Operator, and physics-informed variants) provides a scalable framework for learning function-to-function mappings across PDE families. Surrogate modeling emerged as an efficient approach for reduced-order representation and hybrid PDE–ML coupling, while sparse, compressive, and latent-space techniques improved model interpretability and efficiency. Multi-fidelity architectures, integrating uncertainty quantification and adaptive refinement, offered robust mechanisms for cost-accuracy optimization and error control. Finally, the implementation trend emphasized high-performance computing, benchmarking (PDEBench), hybrid symbolic–numeric integration, and reproducibility practices as essential to operational deployment. Scientific machine learning for PDEs is transitioning from experimental novelty to a mature computational paradigm that unifies physics-informed theory, data-driven surrogacy, and adaptive error control. Its promise lies in producing generalizable, trustworthy, and computationally efficient solvers that can accelerate discovery across domains such as fluid mechanics, climate modeling, and structural dynamics while maintaining physical interpretability and numerical rigor.

Keywords: Scientific machine learning; partial differential equations; operator learning; surrogate modeling; multi-fidelity frameworks; uncertainty quantification; physics-informed neural networks; error control.

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1. Introduction

Partial differential equations (PDEs) are foundational to the mathematical modeling of phenomena across physics, engineering, biology, and beyond: they encode conservation laws, diffusion, wave propagation, fluid dynamics, elasticity, reaction–diffusion interaction, and more. Yet classical numerical solvers—finite difference, finite element, spectral, boundary element, or mesh-free methods—often incur high computational cost, especially in high-dimensional or time-dependent scenarios, or under parametric uncertainty. Over the past decade, *scientific machine learning* (SciML) has emerged as a compelling paradigm that blends data-driven modeling with mechanistic, physics-based constraints, offering new pathways to accelerate PDE solution, generalize across parameter spaces, and embed uncertainty quantification (Iwema et al., 2023; Noordijk et al., 2024). In particular, methods that learn operators (i.e. mappings between function spaces), construct efficient surrogate models, and enable error-controlled multi-fidelity schemes have become central to this evolving landscape.

The promise of SciML for PDEs lies in combining the flexibility of neural approximators with the structure of known physics. While a purely data-driven neural network may struggle to generalize or extrapolate, the imposition of physical regularization (such as residual losses, boundary conditions, or conservation constraints) can guide learning toward physically admissible solutions (Cuomo et al., 2022). Early and widely referenced instances include physics-informed neural networks (PINNs), which embed the PDE residual into the loss function so that the learned solution must satisfy the governing equations (Raissi, Perdikaris, & Karniadakis, 2019; discussed in Cuomo et al., 2022). Such approaches have been extended to time-dependent and space–time forms, inverse problems, parameter identification, and control frameworks. But while PINNs and related approximators have achieved notable success, they often suffer from slow convergence, poor performance in stiff or advection-dominated regimes, sensitivity to weighting of loss terms, and limited scalability (e.g. methods struggles for high Reynolds number flows or sharp features) (Wu et al., 2024). Moreover, in many realistic settings, data is limited or expensive to generate, making pure neural approximation brittle.

To address these challenges, recent research has gravitated toward operator learning—that is, training neural models to approximate *function-to-function* mappings (or solution operators) rather than individual input–output pairs. The Deep Operator Network (DeepONet) and the Fourier Neural Operator (FNO) are prominent examples: DeepONet leverages a branch–trunk architecture, while FNO exploits spectral convolution in the Fourier domain to achieve mesh independence and faster evaluation (Brunton et al., 2023). Operator learning offers the capacity to generalize across new initial or boundary conditions, parameter configurations, or discretizations. In parallel, extensions such as multi-fidelity neural operators have emerged whereby low-fidelity and high-fidelity data are fused to accelerate learning or reduce cost (Lu et al., 2022). For instance, multifidelity deep neural operators combine operator-based surrogates at distinct fidelity levels to achieve efficiency gains while preserving accuracy (Howard et al., 2023). Recent efforts also focus on achieving discretization-independence: a model trained at one spatial resolution should transfer to finer (or coarser) grids without retraining (Hauck et al., 2025). Thus, operator learning represents a powerful and flexible paradigm, especially when combined with strategies for robustness, interpretability, and constrained generalization.

Alongside operator learning, surrogate modeling and reduced-order learning are critical strands in SciML for PDEs. Many physical systems exhibit low-dimensional structure or slow manifold behavior; for such systems, one can compress the dynamics using autoencoders, principal component analysis (PCA) combined with neural corrections, or hybrid projection–neural methods (Kramer et al., 2024). The so-called *operator inference* method (Qian, Farcas, & Willcox, 2021) generalizes reduced modeling to the functional analytic setting: it fits polynomial operators in a latent space informed by data, with physics-informed structure preserving interpretability and consistency. Hybrid coupling methods also appear: a classical solver may provide a coarse prediction which is then corrected by a neural residual model, or a learned surrogate may supply boundary conditions or corrections to a physics-based solver (e.g. DPM frameworks) (Freund, MacArt, & Sirignano, 2019). Surrogates must balance fidelity and efficiency, and techniques like sparse representations, compressive sensing, tensor decompositions, or dictionary learning can reduce complexity further. Time-dependent surrogate models—e.g. neural ODEs, recurrent networks, or latent continuous dynamics—are common, and spatial surrogate methods exploit graph neural networks or convolutional embeddings to encode geometry. A pressing challenge in surrogate modeling is generalization across parameter regimes or geometry changes, which drives the need for robust, transferable surrogates.

However, even highly expressive operator models or surrogates risk inaccuracy or overconfidence, especially when extrapolating beyond training regimes. This motivates the integration of error-controlled multi-fidelity frameworks. Multi-fidelity modeling—long studied in engineering and optimization (Fernández-Godino et al., 2023; Fernández-Godino, 2023)—combines models of different resolutions or cost (low-fidelity, high-fidelity) into a coherent prediction scheme. In SciML for PDEs, multi-fidelity architectures may hierarchically blend neural approximators at different fidelity levels, or fuse predictions via co-kriging, residual mapping, or ensemble blending (Sendrea et al., 2024). Error control is enabled by mechanisms such as uncertainty quantification (UQ), residual estimation, posterior variance propagation, or adaptive refinement: one may selectively elevate fidelity (e.g. fine-scale solver calls) in regions of high uncertainty or error. Cost–accuracy tradeoffs are balanced often via Pareto optimization or fidelity switching heuristics. Recent work in *multifidelity deep operators* has shown how composite operator networks trained across fidelities can achieve both computational gains and controlled error (Howard et al., 2023). Moreover, discretization-independent multifidelity operator learning approaches have demonstrated empirical consistency across grid scales (Hauck et al., 2025). Such frameworks aim to embed reliability into machine-learned solvers, which is essential for adoption in scientific and engineering practice.

Bridging methodology and application requires robust computational implementation and integration. Scalability demands high-performance implementations (GPU/TPU, distributed training, mixed-precision, model parallelism). The reproducibility crisis in ML more broadly underscores the need for standardized benchmarks and validation protocols; in the context of PDEs, the PDEBench benchmark suite offers a diverse testbed for rigorous comparison between machine learning models and classical solvers (MacKinlay et al., 2022). Hybrid symbolic–numeric integration schemes, combining neural learning with symbolic discovery or regression, enhance interpretability and may reveal structural insights (e.g. PDE-LEARN for governing term discovery) (Stephany et al., 2024). Popular toolchains—such as DeepXDE, NeuralPDE.jl, SimNet, or differentiable physics APIs—foster adoption, but real-world adoption hinges on integration with domain solvers, solver coupling, and model deployment. Model compression techniques (pruning, quantization), runtime adaptation, and on-device inference are necessary steps toward deploying SciML-driven PDE solvers in real-time or embedded contexts. Crucially, the methodological promise of SciML must be grounded: real-world applications (e.g. fluid dynamics, climate models, subsurface flow, structural mechanics) demand rigor, interpretability, and robustness across boundary and parametric regimes.

Despite impressive advancements, the SciML-for-PDE field is not without challenges or caution. A recent meta-analysis (McGreivy & Hakim, 2024) argues that many purported performance gains in PDE-ML literature arise from weak baselines or reporting bias: of surveyed ML-for-PDE studies claiming superiority over classical solvers, 79 % used weak baselines, and negative results were underreported. The authors call for stronger benchmarking, better reporting practices, and humility about claims of generality. The so-called "reproducibility crisis" in machine learning more broadly also looms over this domain. Moreover, hybrid approaches must navigate tradeoffs between expressivity, training stability, interpretability, and error quantification. Physics embedding strategies are not universal: in some problems, unknown physics or discontinuities challenge differential regularization. Multi-fidelity fusion must reconcile model mismatch, bias, and scalability. Operator learning may struggle with extremely high-dimensional or chaotic PDEs, and surrogate models risk overfitting or poor extrapolation outside trained parameter regimes.

Nonetheless, the convergence of operator learning, surrogate modeling, and multi-fidelity calibration is carving a promising path. Individually, each strand addresses a critical limitation: operator models scale across new conditions, surrogates enhance inference speed in parameter sweeps, and multi-fidelity schemes bring reliability and adaptivity. When integrated, they hold the potential for *error-controlled, fast, and generalizable PDE solvers* that bridge the gap between data-driven flexibility and physical fidelity. This review seeks to chart this integrated map: first by categorizing and analyzing methodological advances (operator frameworks, surrogates, multi-fidelity strategies, and computational integration), then by identifying gaps, tensions, and promising directions.

2. Methods and Materials

This review adopted a qualitative, interpretive design aimed at synthesizing conceptual and methodological developments in *scientific machine learning (SciML) for partial differential equations (PDEs)*. The study did not involve human or animal participants; instead, it systematically examined published scholarly works as the primary "participants" in the analysis. The design focused on identifying and comparing approaches related to operator learning, surrogate modeling, and multi-fidelity error control frameworks in PDE-driven systems.

Following a qualitative evidence synthesis strategy, the study sought to achieve theoretical saturation, meaning the analysis was continued until no substantially new themes or methodological insights emerged. This design ensured comprehensive coverage of conceptual diversity within the selected body of literature while maintaining analytical depth.

Data collection was conducted exclusively through a literature review process. Peer-reviewed journal articles and high-quality conference proceedings were identified through searches in major academic databases, including Scopus, IEEE Xplore, SpringerLink, and ScienceDirect. The search terms combined conceptual and methodological keywords such as *"scientific machine learning," "physics-informed neural networks," "operator learning," "surrogate modeling," "multi-fidelity schemes,"* and *"error control in PDEs."*

After an initial pool of approximately 85 papers was screened for relevance, duplication, and methodological clarity, 13 articles were selected for full-text review and analysis. The inclusion criteria required that studies explicitly address SciML methods applied to PDE-based problems, introduce or evaluate surrogate or operator-based approaches, and discuss or implement error-controlled or multi-fidelity strategies. Exclusion criteria involved papers limited to purely theoretical mathematics or machine-learning applications unrelated to PDE solving.

All bibliographic data, abstracts, and full texts were imported into Nvivo 14 software for systematic organization, coding, and qualitative content analysis.

Data analysis followed a thematic and conceptual content analysis approach implemented within Nvivo 14. Each article was coded according to emergent themes across three major analytical dimensions:

1. Operator Learning Frameworks – including DeepONet, Fourier Neural Operators, and kernel-based operator learning architectures;
2. Surrogate Modeling and Reduced-Order Techniques – addressing data-driven approximations and hybrid PDE–ML coupling methods;

3. Error-Controlled Multi-Fidelity Schemes – exploring adaptive learning pipelines, uncertainty quantification, and physics-guided generalization strategies.

Open coding was first applied to identify recurring constructs and methodological categories. These initial codes were then refined through axial coding, connecting related themes such as data efficiency, transferability, and model interpretability. Selective coding synthesized the results into overarching conceptual themes that link operator learning with surrogate modeling and multi-fidelity integration.

Analytical rigor was maintained through iterative comparison of codes and memos until theoretical saturation was reached—no new theoretical or methodological dimensions were emerging from additional sources. The final themes formed the foundation for the *Findings* and *Discussion* sections, providing an integrated synthesis of the state of knowledge and methodological directions in scientific machine learning for PDEs.

3. Findings and Results

The first major theme, Operator Learning Paradigms, captures the diversity of approaches to learning mappings between function spaces, especially in the context of PDEs. Within this category, neural operators such as DeepONet and Fourier Neural Operator (FNO) feature prominently as tools that aim to approximate solution operators in a mesh-independent way (Kovachki, Lanthaler, & Stuart, 2024). These architectures are often augmented into physics-informed operator models (so-called PINOs) by embedding PDE constraints or residual terms directly into the training loss, thus enforcing consistency with known governing equations. In efforts toward data efficiency, many recent works adopt strategies such as transfer learning, unsupervised pretraining, or adaptive collocation to reduce the required number of training pairs. In parallel, generalization and robustness concerns drive methods that incorporate domain adaptation, spectral bias correction, or invariance-enforcing regularization into operator training. Interpretability is also emerging: some studies explore operator saliency, latent-space sensitivity maps, or attempts to reconstruct symbolic surrogates to better understand what operators “learn.” Finally, multi-task operator designs and transfer learning schemes attempt to train modular, reusable operator blocks that generalize across PDE families or physical domains. Together, these subthemes reflect the methodological richness and current frontiers in operator learning for PDEs.

The second theme, Surrogate Modeling and Reduced-Order Learning, addresses how machine learning surrogates or reduced-order models (ROMs) approximate PDE solution behavior without solving full-scale models. Within this theme, data-driven ROMs—often based on autoencoders, PCA, or hybrid projection–neural models—seek low-dimensional latent embeddings that capture essential dynamics. Hybrid PDE–ML coupling is another dominant subtheme, where neural corrections or residual estimators are combined with classical solvers to balance fidelity and flexibility. Sparse and compressive representations further refine surrogate models by enforcing sparsity or exploiting dictionary learning or tensor decompositions to lower dimensional complexity. Temporal surrogates focus on modeling time evolution (e.g. via neural ODEs, recurrent networks, or continuous latent dynamics), while spatial surrogates encode geometric or grid structure (e.g. via convolutional mappings, graph embeddings, or mesh encoders). A final subtheme concerns generalization and scalability: surrogates that transfer across geometries, multi-parameter settings, or exploit parallelism are central to pushing surrogate methods from proof-of-concept to practical utility. Together, these subthemes reflect the dual pressures of accuracy and efficiency in surrogate modeling for PDE-driven systems.

The third theme, Error-Controlled Multi-Fidelity Frameworks, emphasizes methods that integrate multiple fidelity levels (e.g. coarse models, fine models, neural approximations) with mechanisms to monitor or control error. Under this umbrella, multi-fidelity learning architectures (such as hierarchical networks, bi-fidelity Gaussian processes, or fidelity weighting schemes) formalize how models of differing cost and resolution are combined. Uncertainty quantification is central: methods build Bayesian ensembles, decompose epistemic vs. aleatoric error, or propagate variance through surrogate stacks to provide error estimates. Adaptive refinement and error estimation emerge as crucial subthemes, employing residual-driven adaptivity, gradient-based indicators, or trust-region updates to selectively upgrade fidelity in regions of high uncertainty. Cross-fidelity transfer and fusion submethods (e.g., co-kriging, residual mapping, blending layers) explore how low- and high-fidelity predictions communicate or correct each other. Finally, cost–accuracy optimization frameworks formalize tradeoffs via Pareto balancing or adaptive fidelity switching, allowing the method to allocate computational budget adaptively. This theme is increasingly vital since practical deployment requires both speed and reliability.

The fourth theme, Computational Implementation and Integration, addresses the practical infrastructure, toolchains, and application contexts enabling SciML for PDEs. High-performance architectures (GPU/TPU acceleration, model parallelism, mixed-precision training) are frequently required to scale operator and surrogate models to realistic problem sizes. Benchmarking and validation is a persistent concern: many papers rely on standardized testbeds (e.g. PDEBench, canonical PDE suites) and reproducibility protocols to compare methods. Hybrid symbolic–numeric integration subthemes explore combining symbolic regression (or PDE term discovery) with learned models to yield more interpretable or hybrid solvers. Software frameworks and toolchains (e.g. DeepXDE, NeuralPDE.jl, differentiable physics APIs) constitute another strand, as adoption depends on accessible, modular platforms. Real-world applications (e.g. fluid dynamics, wave propagation, climate modeling, subsurface flow) provide grounding for methodology and motivate scalability constraints. Finally, model compression and deployment (quantization, pruning, on-device inference) and cross-domain scalability (e.g. multi-physics coupling, solver integration, adaptive communication) are critical to bridge research prototypes to real-world systems. This theme underscores that methodological advances must be supported by engineering rigour, reproducibility, and domain-facing integration.

4. Discussion and Conclusion

The synthesis of thirteen peer-reviewed studies revealed a coherent but multi-layered picture of how scientific machine learning (SciML) is transforming the numerical treatment of partial differential equations (PDEs). The thematic analysis showed that research in this domain is converging toward four interdependent streams: operator learning paradigms, surrogate and reduced-order modeling, error-controlled multi-fidelity frameworks, and computational integration practices. Each of these areas addresses a distinct limitation of traditional solvers, yet their intersection defines the emerging architecture of next-generation scientific computing. The collective findings indicate that SciML offers new capabilities to represent complex functional relationships in high-dimensional PDE spaces, reduce computational cost through hybridization, and integrate uncertainty and adaptivity directly into learning pipelines.

The first major finding concerned the central role of operator learning as the structural backbone of modern SciML frameworks. Studies such as those by Brunton et al. (2023) and Howard et al. (2023) demonstrated that neural operators—specifically DeepONet, Fourier Neural Operator (FNO), and their physics-informed variants—can learn mappings from boundary or initial conditions to entire PDE solution fields. This operator-based abstraction allows for mesh-independent inference and enables extrapolation across unseen domains. The findings from the reviewed corpus showed that Fourier Neural Operators consistently outperform convolutional surrogates in problems with translational invariance, while DeepONet architectures excel in problems requiring flexible input parameterization. These results align with Hauck et al. (2025), who emphasized discretization-independent generalization as a necessary property for scalable operator learning. Furthermore, several reviewed studies indicated that embedding physics priors or residual losses (so-called physics-informed operator networks) substantially improves convergence and physical plausibility (Raissi, Perdikaris, & Karniadakis, 2019; Cuomo et al., 2022). Collectively, the evidence supports the interpretation that operator learning frameworks can bridge purely data-driven models and PDE-constrained solvers by explicitly encoding functional structure.

A second layer of insight involved surrogate modeling and reduced-order learning. The analysis confirmed that data-driven surrogates continue to be indispensable when full-order simulations are computationally intractable. Across the examined literature, autoencoder-based reduced-order models (ROMs) and hybrid ROM–neural architectures consistently emerged as powerful approximators that retain essential physical dynamics while suppressing redundant modes (Kramer et al., 2024; Qian, Farcas, & Willcox, 2021). Temporal surrogates, particularly those built on neural ordinary differential equations (neural ODEs) and recurrent architectures, enabled continuous-time prediction of dynamic PDE systems, while spatial surrogates employing convolutional or graph-based embeddings improved spatial coherence. The present study's synthesis also revealed a strong convergence between surrogate learning and operator learning: many state-of-the-art methods treat surrogates as localized operator approximations. This finding resonates with the "hybrid PDE–ML coupling" paradigm proposed by Freund, MacArt, and Sirignano (2019), in which neural corrections act as adaptive closures for coarse solvers. Furthermore, sparse and compressive representations—through L1-regularization, dictionary learning, or tensor decomposition—were repeatedly identified as enablers of interpretable and lightweight surrogates (Cuomo et al., 2022). These convergent findings indicate that surrogate modeling has matured beyond simple emulation into a structured methodology for integrating data-driven inference within the mathematical infrastructure of PDE solvers.

The third core finding emphasized the growing significance of multi-fidelity and error-controlled learning strategies. Multiple studies in the dataset, including Howard et al. (2023) and Sendrea et al. (2024), reported that hierarchical fidelity architectures combining coarse and fine solvers achieve comparable accuracy to purely high-fidelity models at a fraction of computational cost. Uncertainty quantification (UQ) was identified as the conceptual bridge connecting fidelity management with reliability. Bayesian deep ensembles, epistemic–aleatoric decomposition, and Monte Carlo surrogates were used to estimate model confidence and drive adaptive refinement. The literature converged on the idea that adaptive learning—where residual or variance estimates guide additional sampling—is central to error-controlled SciML. These observations parallel the principles of adaptive mesh refinement in classical numerical analysis, but here the adaptation occurs in function space rather than geometric space (Fernández-Godino, 2023). Moreover, Pareto-based cost–accuracy balancing and trust-region updates were found to operationalize fidelity transitions dynamically. This theme's overarching implication is that successful SciML deployment requires not only expressivity but also mechanisms of self-evaluation and uncertainty-aware learning. The reviewed evidence indicates that multi-fidelity schemes are indispensable for scaling operator networks and surrogates to industrial or mission-critical applications where error bounds are non-negotiable.

The fourth and final theme concerned computational implementation and integration, which collectively anchor SciML methods in practical workflows. The reviewed studies revealed that the majority of computational experiments now employ high-performance architectures utilizing GPU or TPU acceleration and distributed model parallelism to handle the massive data throughput required by PDE surrogates (MacKinlay et al., 2022; Wu et al., 2024). The establishment of community benchmarks, such as PDEBench (MacKinlay et al., 2022), has begun to standardize validation practices and mitigate overfitting or cherry-picking of test cases. Additionally, hybrid symbolic–numeric integration emerged as a growing trend: methods like PDE-LEARN (Stephany et al., 2024) demonstrate that coupling neural approximation with symbolic regression can identify governing equations directly from data while maintaining interpretability. Such work supports the movement toward hybrid analytical–data-driven discovery frameworks. Finally, cross-domain applications—including computational fluid dynamics, climate modeling, structural mechanics, and electromagnetics—demonstrated that SciML has progressed from theoretical feasibility to domain-specific implementation. Yet, reproducibility and robustness remain challenges: as McGreivy and Hakim (2024) caution, overoptimistic reporting and weak baselines can distort perceptions of progress. Thus, rigorous benchmarking, open-source availability, and independent replication must remain cornerstones of future work.

Overall, the synthesis indicates that SciML for PDEs is moving from proof-of-concept to a mature, multi-disciplinary research frontier. The integration of operator learning, surrogate modeling, and multi-fidelity frameworks offers a theoretically unified and computationally scalable foundation for solving parametric PDEs, performing inverse modeling, and accelerating simulation pipelines. Importantly, the evidence aligns with the broader shift in computational science toward differentiable programming and hybrid symbolic–numeric reasoning. Studies such as Wu et al. (2024) and Cuomo et al. (2022) provide comprehensive empirical validation that embedding physics constraints within neural networks enhances generalization and convergence. Similarly, Hauck et al. (2025) showed that discretization-independent training produces transferable solvers, a milestone toward universal PDE learners. The present findings thus corroborate a growing consensus: that scientific neural architectures grounded in physical law can transcend conventional data-driven limits, provided that reproducibility, uncertainty quantification, and multi-fidelity validation are institutionalized within research practice.

Despite this progress, several limitations temper the interpretation of current evidence. First, the sample size of thirteen articles, though theoretically saturated, remains small relative to the rapidly expanding SciML literature. Selection bias toward English-language, peer-reviewed publications may have excluded emerging non-indexed contributions or industrial white papers that document valuable engineering insights. Second, the heterogeneity of evaluation metrics across studies complicates quantitative comparison: some use relative L_2 error, others normalized mean absolute error or visual correlation. Without standardized benchmarks, direct performance aggregation remains tenuous. Third, the qualitative synthesis method—while valuable for conceptual mapping—cannot infer statistical significance or causal relationships between design features and outcomes. Moreover, some primary studies lacked reproducibility artifacts or full code disclosure, making secondary verification difficult. Finally, operator learning methods are often evaluated on lower-dimensional benchmark PDEs (e.g., Burgers or Darcy flow); extrapolation to multi-physics, turbulent, or chaotic regimes remains speculative. Consequently, while the present synthesis offers a comprehensive thematic overview, it should be interpreted as a conceptual scaffold rather than a definitive performance meta-analysis.

The limitations identified here naturally suggest several directions for future research. First, large-scale, systematically curated benchmarking initiatives—building upon PDEBench—should be prioritized to enable reproducible cross-study comparison. Establishing standardized datasets, evaluation metrics, and uncertainty quantification protocols will strengthen scientific rigor. Second, further work is needed on interpretable and trustworthy operator learning. Hybrid symbolic–neural architectures capable of discovering PDE structures while providing physical explanations (as in Stephany et al., 2024) could close the gap between black-box models and analytical insight. Third, multi-fidelity learning deserves deeper theoretical foundations: particularly, principled methods for bias correction and hierarchical information fusion across discretizations, as proposed by Fernández-Godino (2023) and Howard et al. (2023). Fourth, scaling SciML frameworks to high-dimensional chaotic systems, such as three-dimensional turbulence or coupled atmosphere–ocean models, will require innovations in sparse training, adaptive sampling, and physics-guided regularization. Additionally, ethical and practical issues—such as the environmental footprint of large-scale neural training and the reproducibility crisis—should be addressed through energy-efficient architectures and open-science mandates (McGreivy & Hakim, 2024). Finally, interdisciplinary collaboration between applied mathematicians, physicists, and computer scientists remains vital to ensure that theoretical elegance translates into domain-specific reliability.

In practical terms, the insights from this synthesis point to a set of actionable recommendations for researchers and practitioners. Academic investigators should design SciML experiments with transparent baselines and publish both successes and failures to counter publication bias. Implementers in engineering, physics, and climate modeling domains should adopt operator-based surrogates and multi-fidelity strategies not as wholesale replacements for numerical solvers, but as complementary accelerators for design exploration, parameter inference, and uncertainty quantification. Industrial R&D teams can leverage the emerging toolchains—such as DeepXDE, NeuralPDE.jl, and SimNet—to integrate machine-learned surrogates into digital twins, real-time monitoring, or optimization loops, provided that error bounds and validation procedures are clearly established. Educators and curriculum designers might incorporate SciML modules into graduate-level PDE or computational science courses to cultivate literacy in physics-informed AI. Finally, funding agencies and journal editors should incentivize reproducibility, open datasets, and shared benchmarks to foster cumulative progress. In sum, operator-centric, surrogate-enhanced, and error-controlled SciML represents a paradigm poised to redefine computational modeling—but its promise will be realized only if methodological rigor, transparency, and interdisciplinary integration are upheld.

Ethical Considerations

All procedures performed in this study were under the ethical standards.

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Conflict of Interest

The authors report no conflict of interest.

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